

# RESEARCH MEMORANDUM

PRELIMINARY INVESTIGATIONS OF STRENGTH CHARACTERISTICS OF  
STRUCTURAL ELEMENTS AT ELEVATED TEMPERATURES

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SUMMARY

The problem of predicting structural strength of columns, plates, and stiffened panels at uniform elevated temperatures is surveyed. For determining short-time strength, where creep effects are negligible, the material stress-strain curve at the appropriate temperature can be used for columns and plates and an "effective" stress-strain curve may be useful for stiffened panels. For determining the long-time or creep strength, a method requiring a limited amount of test data is given for columns. No theories are available at present for predicting the creep strength of plates or stiffened panels; however, some data obtained from creep tests on stiffened panels at elevated temperatures are included.

INTRODUCTION

The determination of the strength of structural elements at elevated temperatures has become increasingly important because of the aerodynamic heating experienced by aircraft flying at supersonic speeds. As the temperature of the aircraft structure increases, the ability of the structure to support a given load decreases. This decrease in strength is a result of changes in the material properties accompanying temperature increase - notably a lowering of the yield strength and a decrease in the stiffness or modulus of elasticity as well as a decrease in the creep resistance of the material.

As indicated in figure 1 the problem of determining structural strength is actually twofold; namely, the determination of the strength of a member under short-time loading where creep effects do not have to be considered and the determination of the strength under long-time loading where creep within the member must be considered. Some of the methods that are available for determining these strengths for structural elements subject to uniform temperature and which indicate recent theoretical and experimental investigations of the National Advisory Committee

for Aeronautics in regard to this problem are reviewed briefly. Three general types of structural elements - columns, plates, and stiffened panels - are discussed.

## SHORT-TIME STRENGTH

### Columns and Plates

Insofar as the short-time strengths of columns and plates are concerned, past work on plastic buckling has led to a practical solution of the problem (see, for example, ref. 1). Briefly, this work indicates that the stress-strain curve is the key to the calculation of buckling and maximum strength. The short-time strength of columns and plates at elevated temperatures can therefore be calculated by making appropriate use of the stress-strain curve which corresponds to the temperature of the structural element. Figure 2, for example, shows the result of such calculations for columns. This figure shows a plot of buckling stress against slenderness ratio  $L/\rho$  for columns made of 75S-T6 aluminum alloy. The solid lines represent the critical or tangent-modulus buckling stresses for the columns at six different temperatures ranging from room temperature to 600° F. The dashed-line curves show the buckling stresses for the columns as determined by the Euler column formula. The stress-strain curve for the material at each temperature shown was used to obtain the values of tangent modulus necessary in computing the column curves. The maximum loads that can be supported by columns, determined from actual tests, are, in general, in good agreement with these calculated results.

The buckling stress of plates at elevated temperatures can similarly be determined from the stress-strain curve, and the buckling stress so determined is, for practical purposes, also the average maximum stress if it exceeds three-fourths of the compressive yield stress. If the buckling stress is less than three-fourths of the yield stress, then the maximum strength in many cases can be computed from a curve like that of figure 9 of reference 1, obtained at room temperature but applicable at elevated temperature when used in conjunction with the appropriate value of the compressive yield stress.

### Stiffened Panels

A slightly different approach has been found to be successful for determining the short-time strength of stiffened panels. In the case of columns and plates, the stress-strain curve for the material at the appropriate temperature was used to determine the maximum load for

the member. For stiffened panels, an "effective" stress-strain curve is useful. This effective stress-strain curve is obtained from a compression test of a short stiffened panel. The results of such a test are shown in figure 3. The solid line is the result of a test at 400° F on a stiffened panel made of 24S-T3 aluminum alloy and shows average stress plotted against unit shortening. The dashed line represents the material stress-strain curve at the same temperature. The difference between these two curves shows the effect of local buckling. The panel tested was a short panel with a slenderness ratio  $L/\rho$  of 20. The slopes of this effective stress-strain curve are used in preference to the slopes of the material stress-strain curve for determining values of the tangent-modulus. With this information, the maximum loads for panels of identical cross section but of longer lengths can be determined from the generalized Euler column formula. Figure 4 shows a comparison between calculated maximum stresses for stiffened panels and experimentally determined maximum stresses. The solid-line curves were obtained by making use of the effective stress-strain curve for the panel at the appropriate temperature. The test points shown are the maximum stresses determined from laboratory tests on stiffened panels made of 24S-T3 aluminum alloy. All panels were tested flat-ended with a fixity coefficient of 3.75 as assumed in all flat-ended panel tests at the NACA. Figure 4 shows that satisfactory agreement exists, in general, between the calculated maximum stresses and the experimentally determined maximum stresses for the panels at both of the temperatures indicated.

## LONG-TIME STRENGTH

### Columns

In the study of long-time or creep strength of structural components, the most work, both theoretical and experimental, has so far been done on columns. A theoretical analysis was made at the NACA of the creep behavior of an idealized H-section column under constant temperature and constant load (ref. 2). This work has recently been extended to the solid-section column (ref. 3).

In the analysis of the creep strength of a solid column, a material creep law was selected in the form which follows (see fig. 5):

$$\epsilon = \frac{\sigma}{E} + Ae^{B\sigma_t K} t$$

where  $\epsilon$  is the total strain,  $\sigma$  is the applied stress for the creep test, and  $t$  is the time after application of the stress. The symbols  $A$ ,  $B$ , and  $K$  designate material constants whose values depend on the temperature and  $E$  is Young's modulus, which is also a function of the temperature. The total strain is composed of two parts - an elastic part which results immediately upon application of the stress and a time-dependent part. This type of creep equation was suggested by Battelle Memorial Institute for 75S-T6 aluminum alloy at 600° F and since has been found to hold approximately for this material at other temperatures and for at least one other material - a low-alloy steel at 800° F and 1100° F. The type of creep curve which this creep relation implies is shown in figure 5. The exponent  $K$ , whose numerical value is less than 1, causes the creep curves to be concave downward. If the value of  $K$  were 1, the creep curves would be straight lines. The curves shown approximate those for 75S-T6 aluminum alloy at 600° F, for which  $K \approx 2/3$ .

Shanley's creep hypotheses were used as a basis of generalizing the creep law to cover stress varying with time. The usual assumptions which permit the use of elementary beam theory were made regarding the column. In addition, it was assumed that the column was pin-ended, that the initial curvature was in the form of one-half a sine wave, that the deflected shape always remained sinusoidal but the amplitude increased with time, and that the load was applied rapidly enough so that negligible creep occurred during the loading period but not so rapidly that dynamic effects had to be considered. The equations resulting from the analysis of the solid column are very difficult to solve, but useful parameters have been obtained from these equations. These parameters will be shown later in connection with experimental test results.

The experimental study of the creep strength of columns at the NACA was confined to short-time creep tests which lasted a few hours rather than hundreds of hours. The purpose of the experimental study was two-fold - to obtain design data as well as to correlate the data with the parameters of the previously mentioned theory. The columns were made of 75S-T6 aluminum alloy, which was selected because the creep law previously shown was found to apply to this material. The columns were tested in a pin-ended condition at temperatures ranging from 300° F to 600° F. The midheight deflection was recorded autographically against time during the creep test. A typical set of these deflection data is shown in figure 6. This figure shows midheight deflection  $d$  as a fraction of the column thickness  $b$  plotted against time for two different columns tested at 300° F. The initial out-of-straightness at the midheight  $d_0/b$ , the applied stress, and the slenderness ratio  $L/\rho$  are all indicated. The highest point of each curve corresponds to collapse of the column. It is interesting to note that the lateral deflection immediately prior to collapse of the column is still a small

percentage of the column thickness and that collapse is rather sudden. This fact is useful because it indicates that a small-deflection analysis should be valid, insofar as deflections are concerned, over practically the entire lifetime.

The most significant information from this type of plot is the column lifetime. The information on column lifetime has been abstracted from forty curves of this type. A portion of these results (that portion which is for a temperature of  $600^{\circ}$  F) is illustrated in figure 7. The parameters used in this plot were obtained from the theoretical analysis previously mentioned. The lifetime parameter is a combination of three quantities; namely, the actual lifetime of the column  $t_{cr}$ , expressed in hours, the material creep constants  $B$  and  $K$  previously mentioned, which, at  $600^{\circ}$  F, have the values  $0.00192/\text{pounds per square inch}$  and  $2/3$ , respectively, and the average stress  $\bar{\sigma}$  in pounds per square inch applied on the column. The crookedness parameter is composed of two quantities which are the measured initial out-of-straightness of the column at the midheight  $d_0$  divided by the column thickness  $b$  and the average stress  $\bar{\sigma}$  applied on the column. The theory predicts that, for a given ratio of the average applied stress to the Euler buckling stress, a single curve should be obtained of lifetime parameter against crookedness parameter. Each of the dashed curves which have been drawn on the basis of test data represents a different value of this stress ratio ranging from 0.1 to 0.9 as shown. The dashed curves indicate that when the ratio of the applied stress to the Euler buckling stress  $\bar{\sigma}/\sigma_E$  decreases, the lifetime of the column increases.

Also, when the column out-of-straightness increases, the lifetime decreases. Plots similar to the one in figure 7 were obtained from the creep tests on columns at other temperatures. This type of plot can be used directly for determining the actual lifetime of columns for any out-of-straightness and applied stress. For example, if values are fixed for out-of-straightness and desired lifetime and a value of applied stress is selected, the stress ratio  $\bar{\sigma}/\sigma_E$  is determined. This stress ratio can then be solved for the slenderness ratio  $L/\rho$ . Figure 8 shows the result of such calculations. This figure shows the lifetime of 75S-T6 aluminum-alloy columns at  $600^{\circ}$  F. The out-of-straightness  $d_0/b$  for the columns is 0.01 where  $d_0$  is the measured initial out-of-straightness at the midheight and  $b$  is the column thickness. The zero time curve, which represents the tangent-modulus stress for the columns, was obtained from the material stress-strain curve. The remaining curves showing lifetime were obtained by cross-plotting the information shown in figure 7. Test data show good agreement with these curves. Similar plots for any other ratio of  $d_0/b$  can be prepared from figure 7 which shows the column lifetime parameter plotted against the column crookedness parameter.



### Plates

It is believed that no theoretical work has been published on the creep behavior of plates. Test data also are lacking on this subject. However, some qualitative remarks can be offered on what might be expected of a plate supported along its four edges when subjected to edge compression sufficient to cause creep. As was found to be the case for columns, initial imperfections should play an important part in determining the lifetime of such a plate provided the plate is not seriously buckled by the applied load. The shape of the initial imperfection as well as its magnitude may have to be considered. When the plate is loaded initially with a load greater than the buckling load, the size of the buckles will probably mask the effect of the relatively small initial imperfections. This latter conjecture is supported to some extent by creep tests performed by the NACA not on plates as such but on stiffened panels.

### Stiffened Panels

The creep tests were performed on stiffened panels made of 24S-T3 aluminum alloy. The tests for all panels were designed to be short-time creep tests. The load applied in every case was sufficiently high to cause some visible buckling of the panel skin. Some of the data obtained from these tests are presented in figures 9 and 10. Figure 9 shows creep-test data for three panels of the same length and cross section. The results are shown in terms of unit shortening plotted against time. The average stress applied on each panel is shown absolutely and also as a percentage of the maximum short-time strength obtained by rapid loading of an identical panel. Collapse of two of the panels occurred when the unit shortening reached approximately 0.9 percent. The short dashed lines indicate the point at which accelerated shortening began. Note that for the two panels tested to failure, the accelerated shortening began at the same value of unit shortening, in this case, 0.006. The results of two of these creep tests are shown in figure 10 in conjunction with the rapid-loading strength test of an identical panel. The result of a rapid loading test is given by the solid curve which shows stress plotted against unit shortening. The results of the creep tests from figure 9 are shown on horizontal lines of figure 10 which represent the stress level at which the panels were tested. The vertical tick marks show the amount of unit shortening which occurred every 30 minutes during the creep test. The circles represent the point at which the creep rate or rate of unit shortening began to increase. Note that the amount of unit shortening corresponding to the start of accelerated creep is equal to the unit shortening corresponding to the maximum stress obtained in the rapid-loading test. This observation was made for all panels tested. Also note that, for the

panels subject to creep, failure or collapse occurred at a unit shortening which was somewhat less than the unit shortening corresponding to the same stress level obtained in the unloading portion of the rapid-loading test.

Few conclusions can be made as yet from this preliminary investigation of creep strength of panels. However, if additional test data substantiate the trends shown, an approximate solution may be developed which will be helpful for predicting the long-time or creep strength of panels.

#### CONCLUDING REMARKS

In conclusion, it can be said that the short-time maximum strength of members such as columns, plates, and stiffened panels can be predicted with satisfactory accuracy at any temperature. The stress-strain curve for the material at the appropriate temperature is needed for the column or the plate. For the stiffened panel, the effective stress-strain curve is needed. In the field of long-time strength, the situation is not so satisfactory. Insofar as columns are concerned, the theory presented in NACA TN 2956 offers considerable promise for predicting long-time strength. For plates and stiffened panels, however, no theory has as yet been developed for predicting creep strength. Both theoretical and experimental work in this direction will be necessary before a practical solution is found.

Langley Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
Langley Field, Va., April 28, 1953.

#### REFERENCES

1. Heimerl, George J., and Roberts, William M.: Determination of Plate Compressive Strengths at Elevated Temperatures. NACA Rep. 960, 1950. (Supersedes NACA TN 1806.)
2. Libove, Charles: Creep Buckling of Columns. Jour. Aero. Sci., vol. 19, no. 7, July 1952, pp. 459-467.
3. Libove, Charles: Creep-Buckling Analysis of Rectangular-Section Columns. NACA TN 2956, 1953.



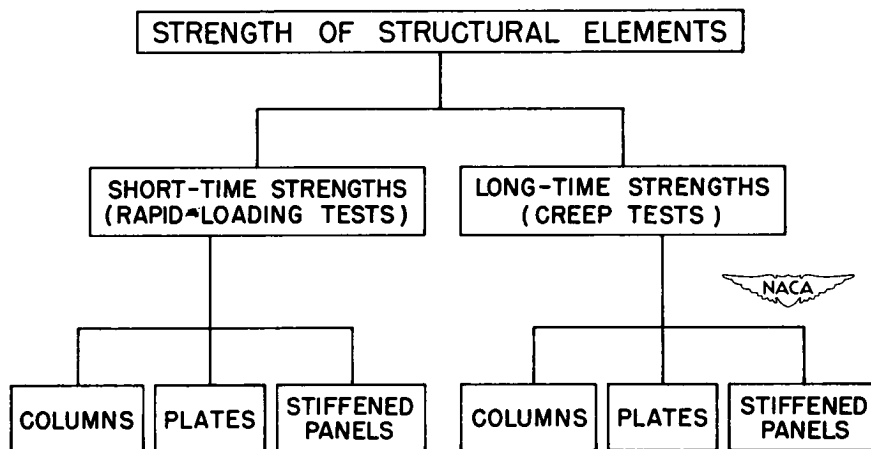


Figure 1.- Determination of structural strength at elevated temperatures.

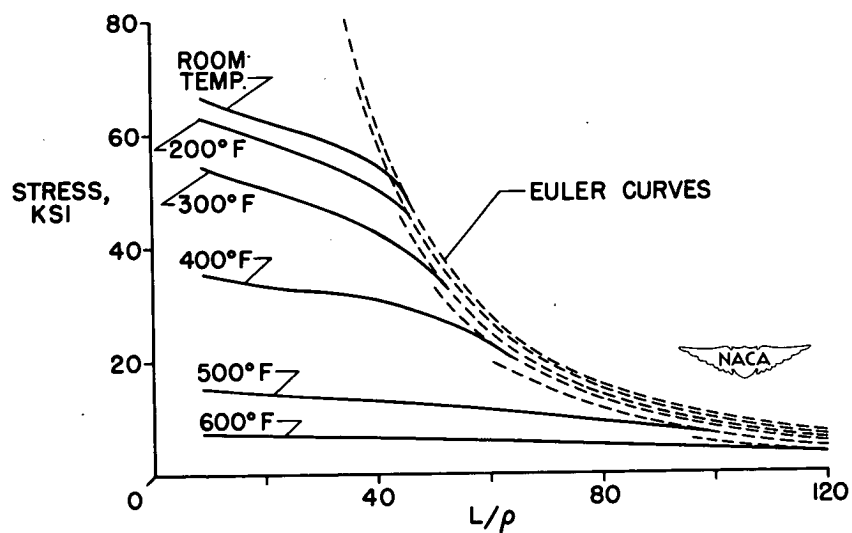


Figure 2.- Tangent-modulus buckling stresses for 75S-T6 aluminum-alloy columns.

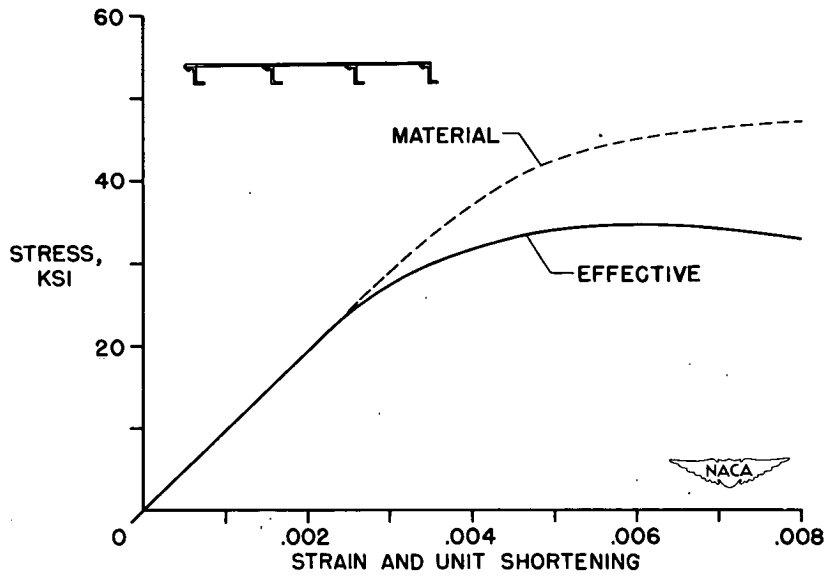


Figure 3.- "Effective" stress-strain curve at 400° F for stiffened panel made of 24S-T3 aluminum alloy.

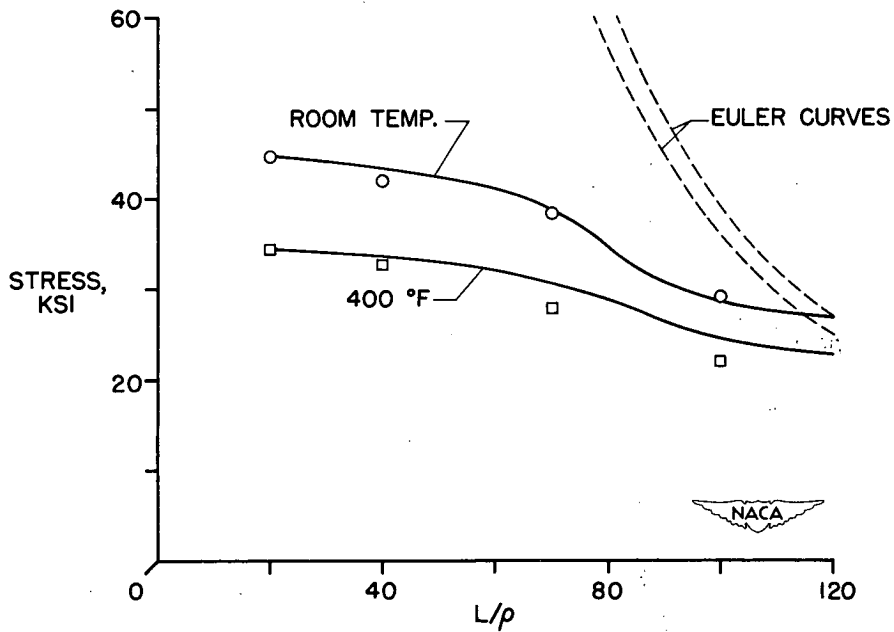


Figure 4.- Maximum stresses for stiffened panels made of 24S-T3 aluminum alloy.

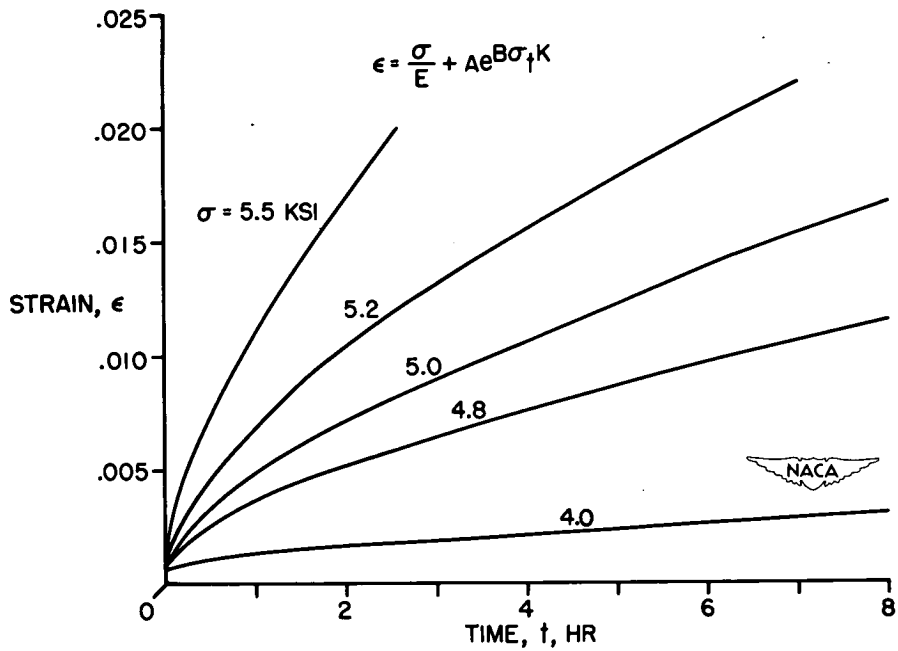


Figure 5.- Theoretical creep curves for 75S-T6 aluminum alloy at 600° F.

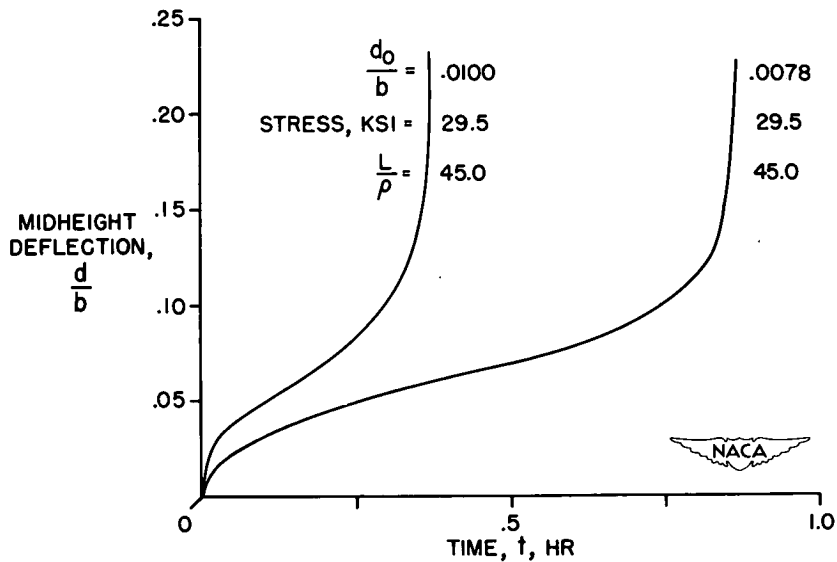


Figure 6.- Creep-test results for 75S-T6 aluminum alloy columns at 300° F.

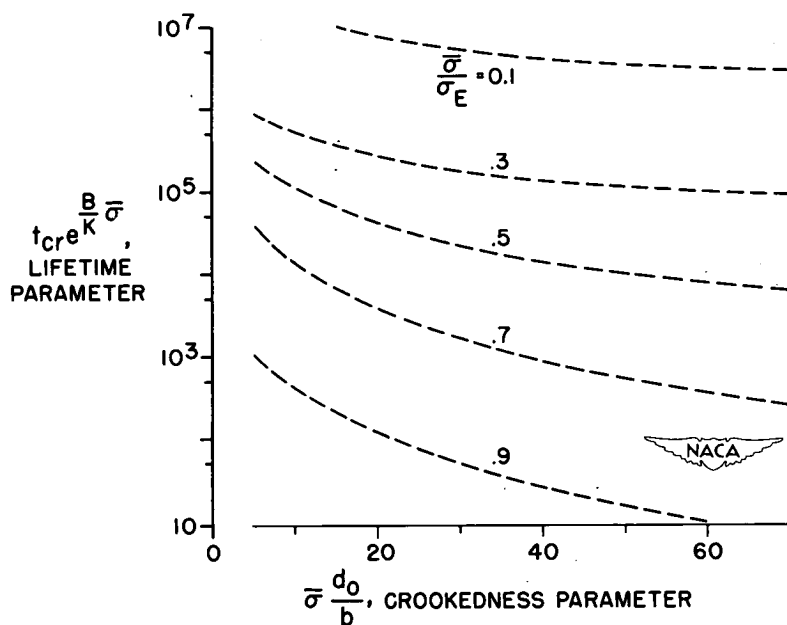


Figure 7.- Lifetime parameter against crookedness parameter for 75S-T6 aluminum-alloy columns at 600° F.

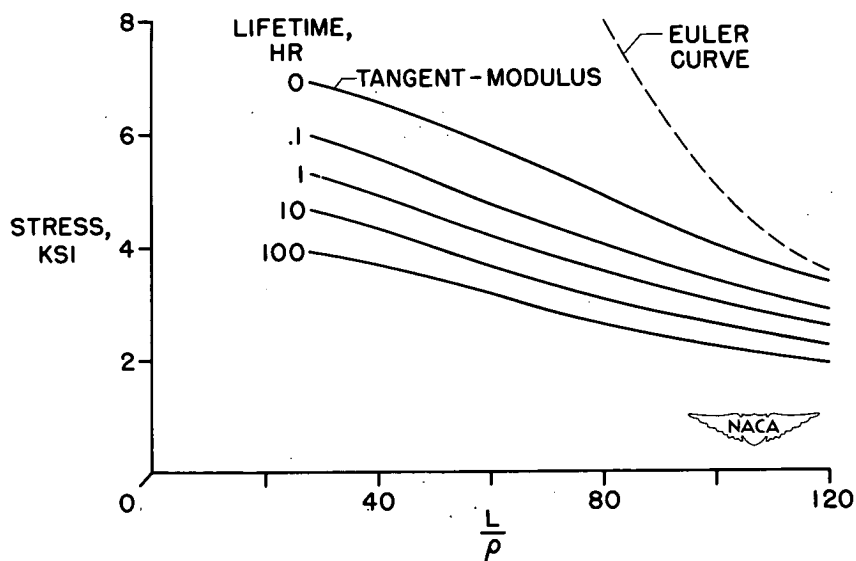


Figure 8.- Creep strength of 75S-T6 aluminum-alloy columns at 600° F.  
 $d_0/b = 0.01$ .

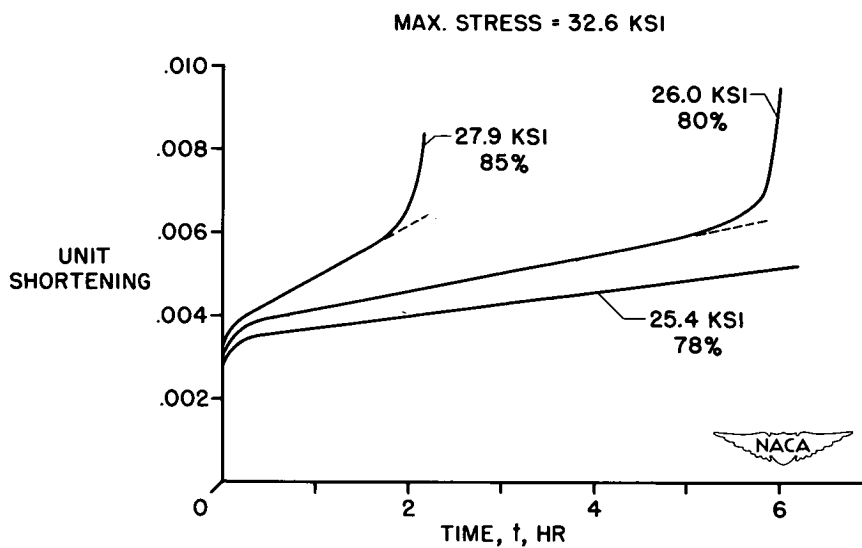


Figure 9.- Creep-test results at 400° F for stiffened panels made of 24S-T3 aluminum alloy.

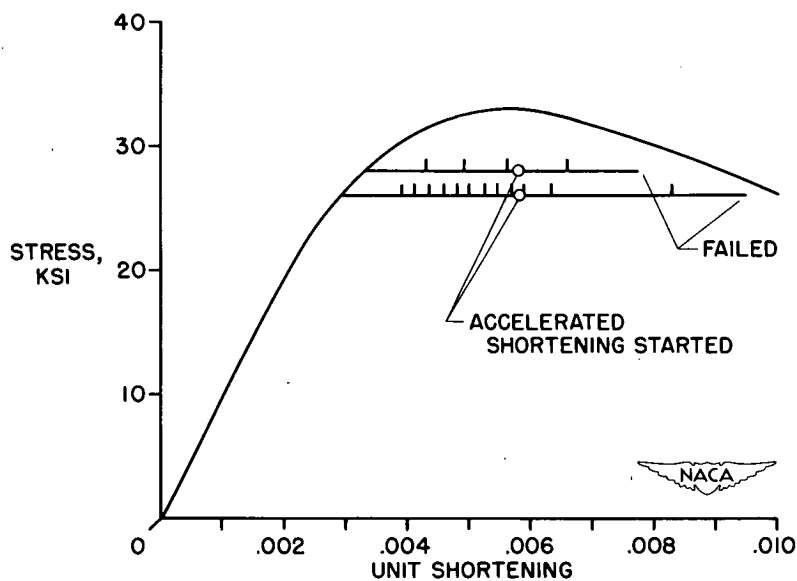


Figure 10.- Short-time and long-time test data at 400° F of stiffened panels made of 24S-T3 aluminum alloy.